DSAβDS 2012 Lisbon

Developing Ecological Models:

Modelling the population dynamic of Spruce Budworm (*Choristoneura* sp.) and its effect on the North American Spruce forests

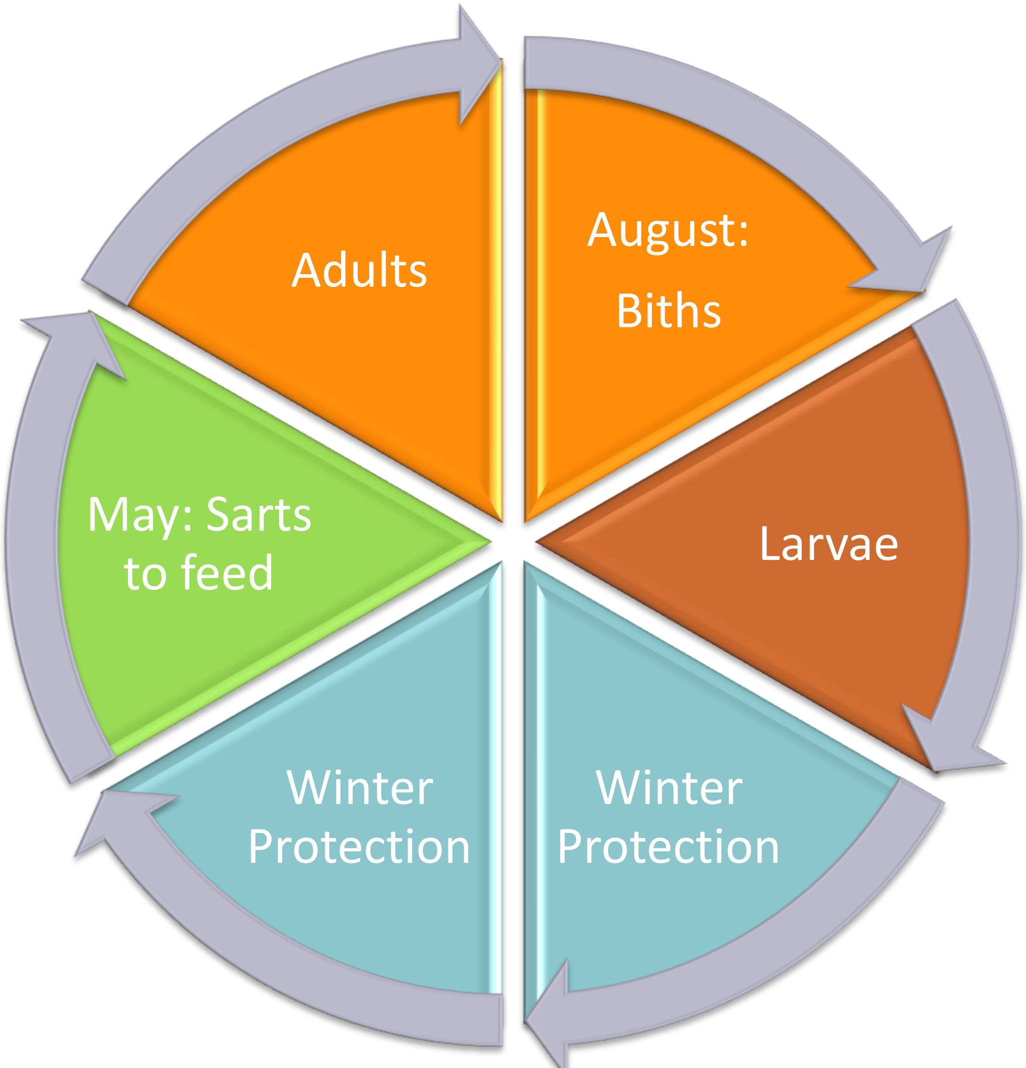
Filipe Rocha, Maíra Aguiar & Nico Stollenwerk

CMAF
Centro de Matemática e Aplicações Fundamentais

Spruce Budworm

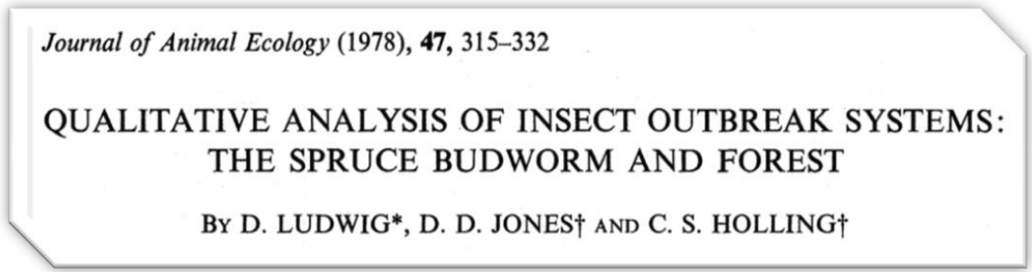
- *Choristoneura* sp.;
- Distributed in North America and Eurasia.
- Parasite of conifers;
- Larvae feed leaves and new tissues;
- The bloom of insects can origin the death of the trees:
  - 1909: Canada;
  - 1922: USA.

Life Cycle


Adults
August:
Biths
Larvae
Winter Protection
Winter Protection
May: Sarts to feed

Model Building

1. Density of Budworms;
2. Branch Surface Area;
3. Energy Reserve;
4. Pesticide Effect.


Journal of Animal Ecology (1978), **47**, 315–332

QUALITATIVE ANALYSIS OF INSECT OUTBREAK SYSTEMS: THE SPRUCE BUDWORM AND FOREST

BY D. LUDWIG*, D. D. JONES† AND C. S. HOLLING†

1. Density of budworms

$$\frac{dB}{dt} = r_B \cdot B\left(1 - \frac{B}{K_B}\right) - \beta \cdot \frac{B^2}{\alpha^2 + B^2}$$

- B – Density of budworms;
- r_B - Growth rate of Budworms;
- K_B – Carrying capacity of Budworms;
- β – Maximum Budworms predated;
- α - Half maximum density for predation.

1. Density of Budworms

Stationary States:

$$B_1^* = 0$$

$$r_B \left(1 - \frac{B}{K_B}\right) - \beta \frac{B^2}{\alpha^2 + B^2} = 0$$

1. Density of Budworms

Variables Change:

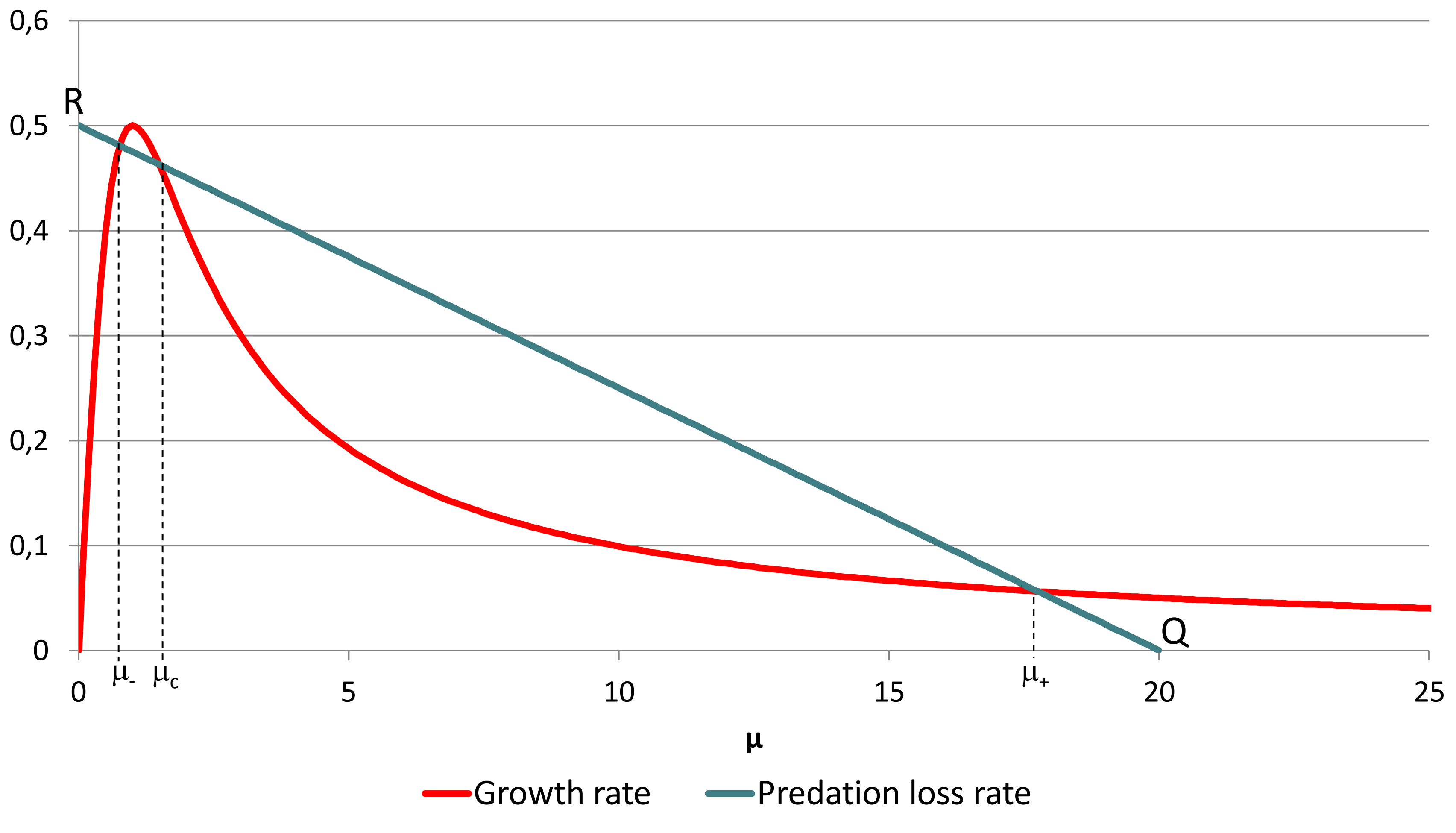
$$\mu = \frac{B}{\alpha'}$$

$$R = \frac{\alpha \cdot r_B}{\beta}$$

$$Q = \frac{K_B}{\alpha}$$

$$R\left(1 - \frac{\mu \cdot r_B}{Q}\right) = \frac{\mu}{1 + \mu^2}$$

1. Density of Budworms



2. Branch Surface Area

$$\frac{dS}{dt} = r_S \cdot S \cdot \left(1 - \frac{S}{K_S} \cdot \frac{K_E}{E}\right)$$

- S – Branch surface area;
- r_S – Branch growth rate;
- K_S – Maximum branch growth.

3. Energy Reserve

$$\frac{dE}{dt} = r_E \cdot E \cdot \left(1 - \frac{E}{K_E}\right) - P \cdot \frac{B}{S}$$

- E – Energy reserve;
- r_E – Energy Reserving rate;
- K_E - Maximum energy level;
- P – Consumption rate of Energy;
- B/S – Stress caused by consumption of foliage.

Branch Surface Area and Energy Reserve

Stationary States:

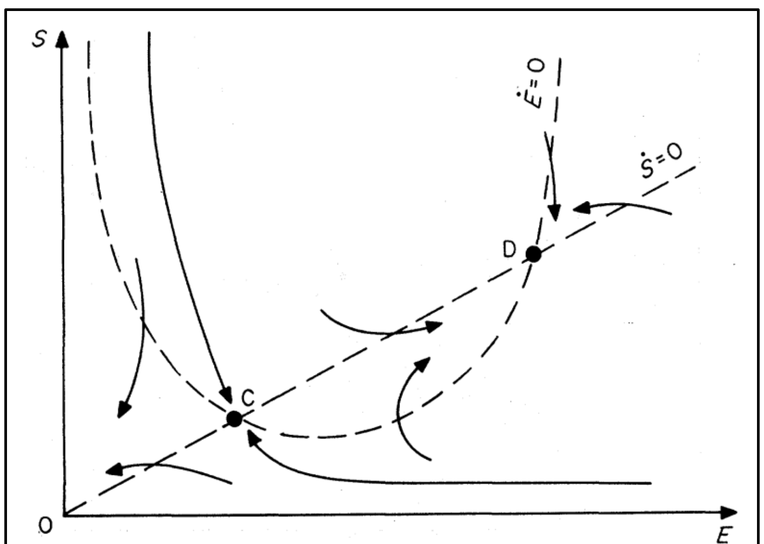
$$S_1^* = 0 \qquad S_2^* = \frac{E \cdot K_S}{K_E}$$

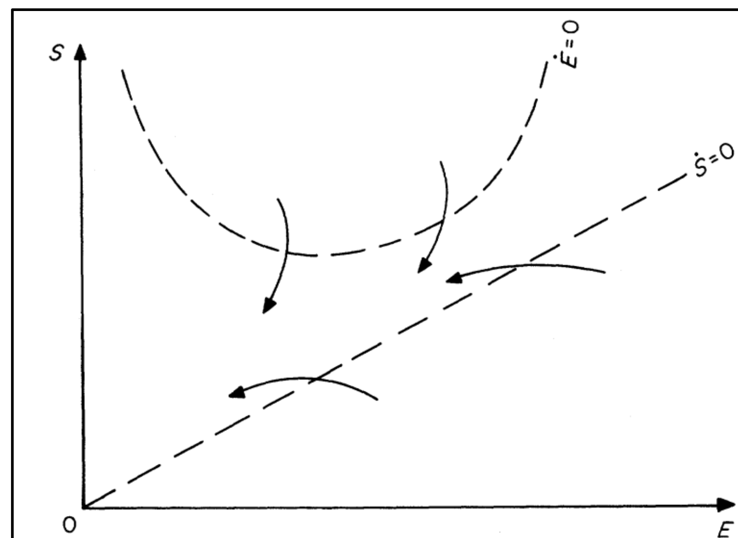
$$S^* = \frac{P \cdot B}{r_E \cdot E \cdot \left(1 - \frac{E}{K_E}\right)} = \frac{P \cdot B \cdot K_E}{r_E} \cdot \frac{1}{E \cdot (K_E - E)}$$

Energy Reserve

- Stationary States:

$$S_2^* = \frac{E \cdot K_S}{K_E} \qquad S^* = \frac{P \cdot B}{r_E \cdot E \cdot \left(1 - \frac{E}{K_E}\right)} = \frac{P \cdot B \cdot K_E}{r_E} \cdot \frac{1}{E \cdot (K_E - E)}$$





(Ludwig, 1978)

Parameters

- It is more correct if the model is building considering the branch area, insted of acre of forest.
- The carrying capacity depends on the bounded area by branch:

$$K_B = K' \cdot S$$

- And it also depends on the energy of the trees:

$$K_B = K' \cdot S \cdot \frac{E^2}{T_E^2 + E^2}$$

  - K' – Carrying capacity in larvae/branch;
  - T_E – Energetic critic limit for the survival of the trees.

Parameters

- The predation are also dependent of the branch surface:

$$\alpha = \alpha' \cdot S$$

  - α' – half saturation for Budworms larvae per unit of branch area.

- The K_B can be substituted in equations of R and Q:

$$R = \frac{\alpha' \cdot r_B}{\beta} \qquad Q = \frac{K'}{\alpha'}$$

- The P values depend on the energy reserve:

$$P = P' \cdot \left(\frac{E^2}{T_E^2 + E^2} \right)$$

Continuous Model

$$\frac{dB}{dt} = r_B \cdot B \left(1 - \frac{B}{K' \cdot S} \cdot \frac{T_E^2 + E^2}{E^2}\right) - \beta \cdot \frac{B^2}{(\alpha' \cdot S)^2 + B^2}$$

$$\frac{dS}{dt} = r_S \cdot S \cdot \left(1 - \frac{S \cdot K_E}{E \cdot K_S}\right)$$

$$\frac{dE}{dt} = r_E \cdot E \cdot \left(1 - \frac{E}{K_E}\right) - P' \cdot \frac{B}{S} \cdot \frac{E^2}{T_E^2 + E^2}$$

Time Dependent Solution

$$B_{n+1} = B_n + \Delta t \cdot \left(r_B \cdot B_n \left(1 - \frac{B_n}{K' \cdot S_n} \cdot \frac{T_E^2 + (E_n^2)^2}{E_n^2} \right) - \beta \cdot \frac{B_n}{(\alpha' \cdot S_n)^2 + B_n^2} \right)$$

$$E_{n+1} = E_n + \Delta t \cdot \left(r_E \cdot E_n \cdot \left(1 - \frac{E_n}{K_E} \right) - P' \cdot \frac{B_n}{S_n} \cdot \frac{E_n^2}{T_E^2 + E_n^2} \right)$$

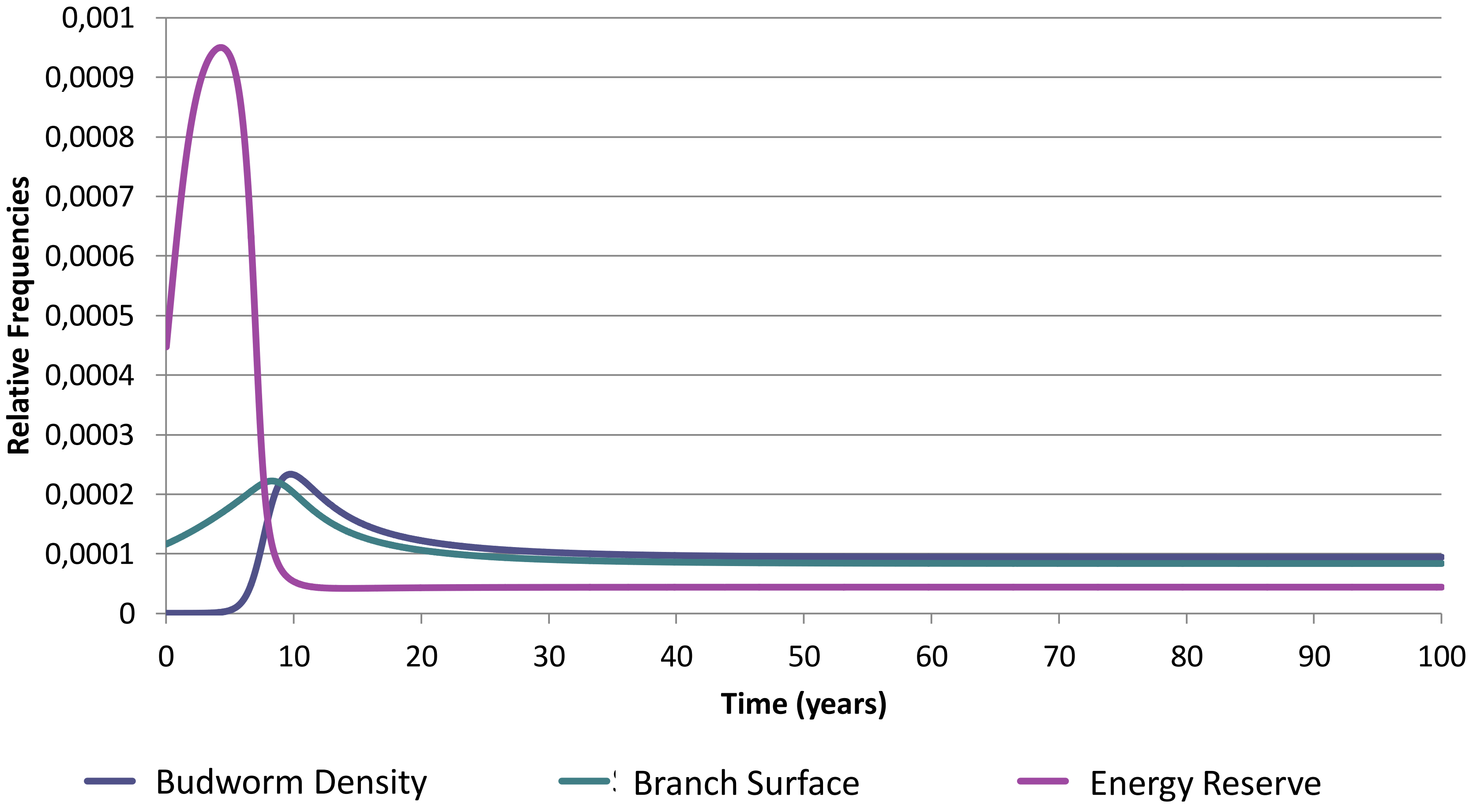
$$S_{n+1} = S_n + \Delta t \cdot (r_S \cdot S_n \cdot \left(1 - \frac{S_n \cdot K_E}{E_n \cdot K_S} \right)$$

Parameters

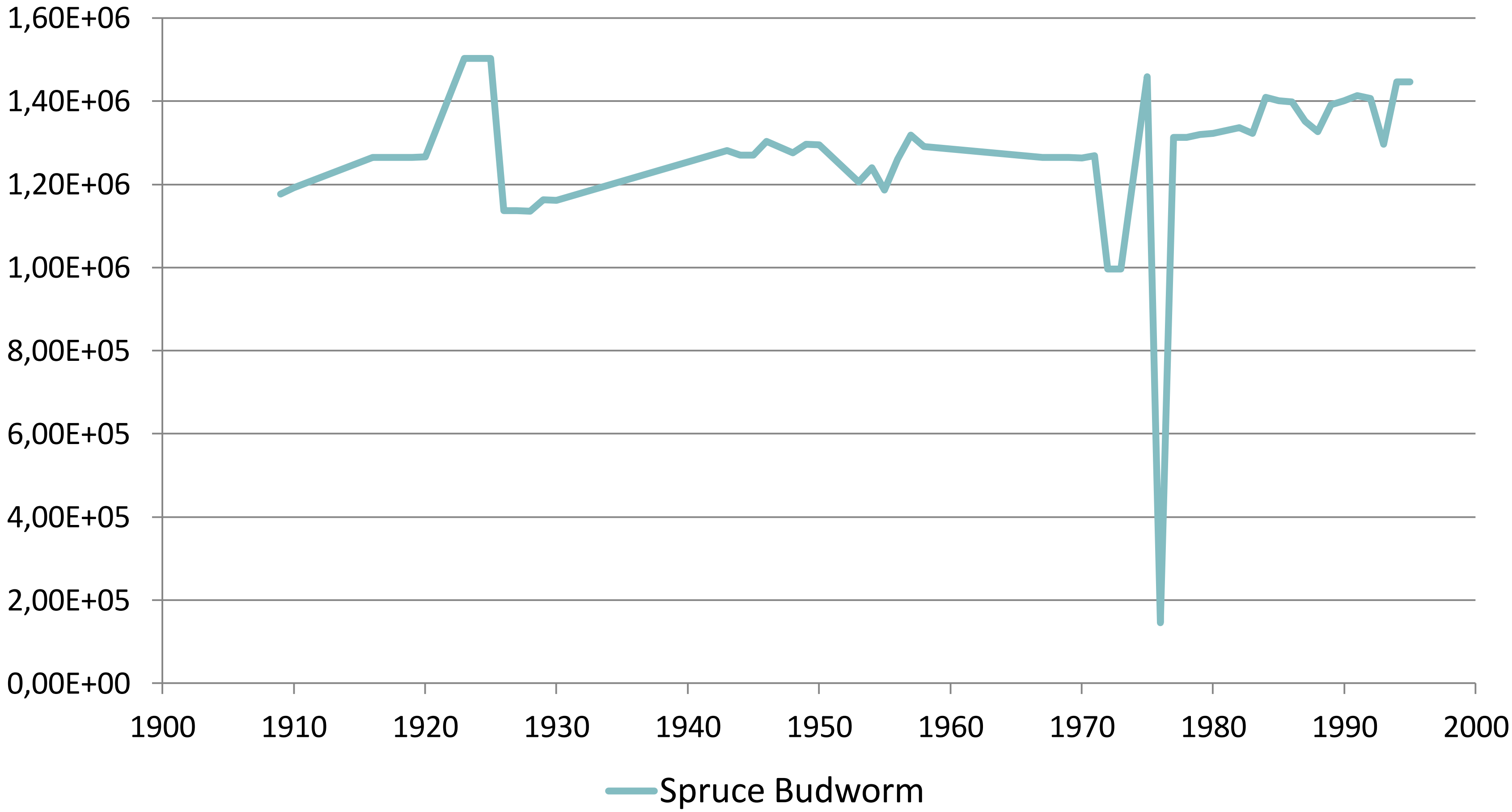
Symbol	Description	Units	Level II	Level III
rB	Budworms growth rate	/year	1.6	1.52
K’	Maximum budworm density	Larvae/branch	100-300	355
β	Maximum budworm predated	Larvae/acre/br anch	20 000-36 000	43 200
α’	Half maximum density for predation	Larvae/branch	1-2	1.11
r_S	Branch growth rate	/year	0.15	0.095
K_S	Maximum brach density	Branch/acre	24 000	25 440
K_E	Maximum energy level	-	1.0	1.0
r_E	Energy growth rate	/year	1.0	0.92
P’	Consumption rate of energy	/larvae	0.0015	0.00195
R	$\alpha' r_B S / \beta$	-	1.07-3.84	0.994
Q	K' / α'	-	50-300	302

(Ludwig, 1978)

Ploting the Model



Real Data of Budworms



(Natural Resources Canada)

Incorporation of the Pesticide

- Equation of the incorporation of the pesticide:

$$D(x) = d^{-wt} \cdot B$$

  - d – Efficiency of pesticide;
  - wt – resistance rate.

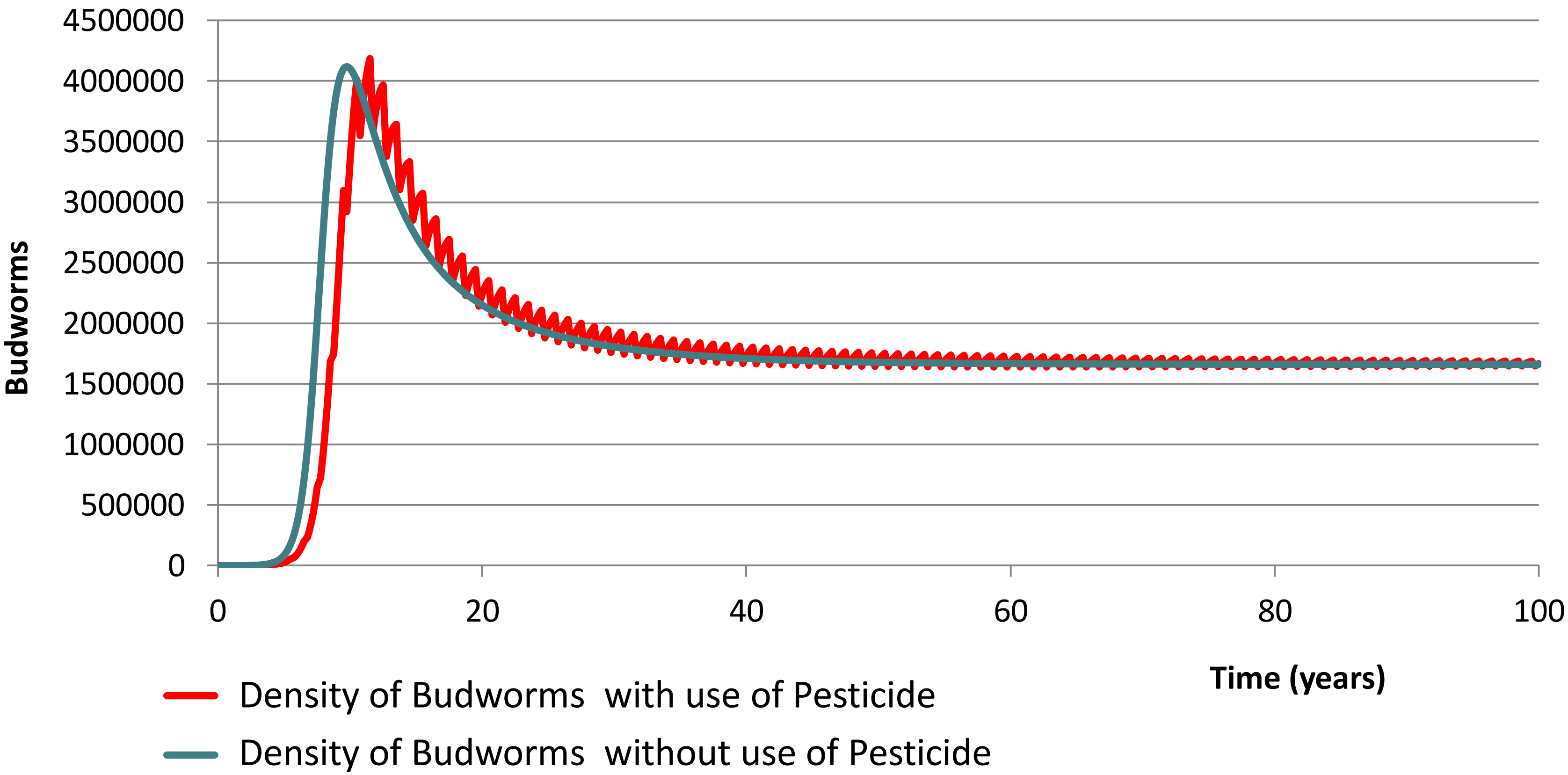
- Model with the use of pesticide:

$$\frac{dB}{dt} = r_B \cdot B \left(1 - \frac{B}{K' \cdot S} \cdot \frac{T_E^2 + E^2}{E^2}\right) - \beta \cdot \frac{B^2}{(\alpha' \cdot S)^2 + B^2} - d^{-wt} \cdot B$$

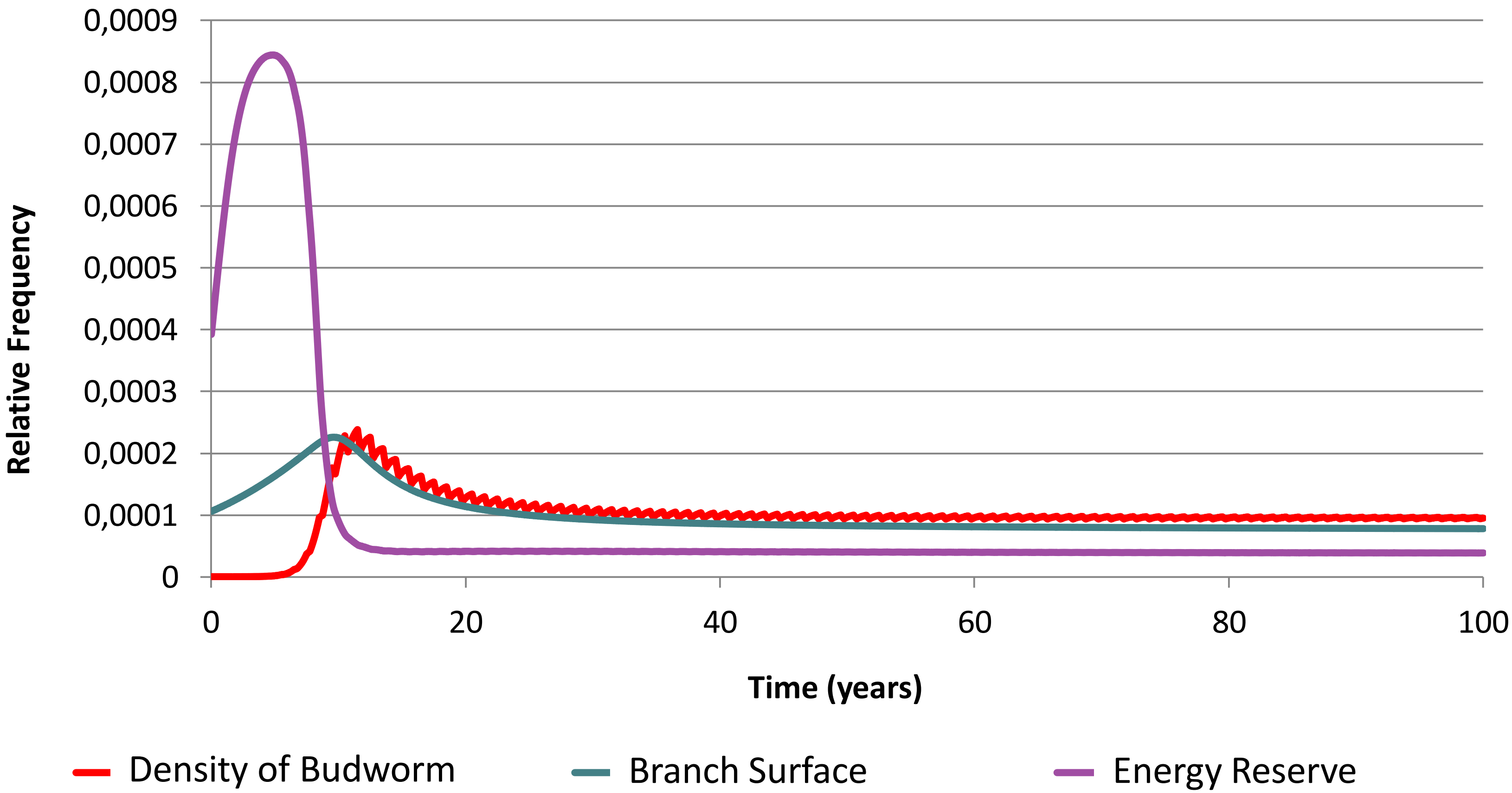
- Time Dependent Solution:

$$B_{n+1} = B_n + \Delta t \cdot \left(r_B \cdot B_n \left(1 - \frac{B_n}{K' \cdot S_n} \cdot \frac{T_E^2 + (E_n^2)^2}{E_n^2}\right) - \beta \cdot \frac{B_n}{(\alpha' \cdot S_n)^2 + B_n^2} - d^{-wt} \cdot B\right)$$

Incorporation of the Pesticide



Incorporation of the Pesticide



Conclusion

- The evolution of budworm's population;
- Prediction about the damages in case of a bloom of insects;
- Effects of possible mitigation measures;
- Predictions about the behaviour of the ecosystem in case of a bloom;

- The models are an important tool to understand the ecosystem and allow the decision makers to know how the system will behave in deferent characteristics.

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Thank you for your interest!